On the $L$-fuzzy generalization of Chu correspondences

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Abstract

In this position paper, we focus on the framework of Chu correspondences extending Mori’s approach [16] to formal concept analysis by proposing suitable definitions of the required concepts in an $L$-fuzzy environment.

1 Introduction

Since its introduction in the eighties, formal concept analysis [9] has become an important and appealing research topic both from the theoretical perspective [18, 11, 21, 3] and from the applicative one. Regarding applications, we can find papers ranging from ontology merging [7, 17], to applications to the Semantic Web by using the notion of concept similarity [8], and from processing of medical records in the clinical domain [10] to the development of recommender systems [5].

Soon after the introduction of “classical” formal concept analysis, a number of different approaches for its generalization were introduced and, nowadays, there are works which extend the theory with ideas from fuzzy set theory [2, 14] or fuzzy logic reasoning [6, 1] or from rough set theory [19, 13, 22] or some integrated approaches such as fuzzy and rough [20], or rough and domain theory [12].

In this paper we concentrate on the categorical approach to formal concept analysis developed in [16], in which the notion of Chu correspondences between formal contexts is introduced. In that paper, the construction of formal concepts associated to a crisp relation between objects and attributes is shown to induce a functor from the category of Chu correspondences to the category of sup-preserving maps between complete lattices. It turns out that the category of Chu correspondences has a *-autonomous category structure which is preserved by the induced functor. Our main contribution here is the development of the initial notions in order to extend the theory of Chu correspondences to an $L$-fuzzy framework.
2 Preliminaries

We will assume that the reader is familiar with the standard notions of classical formal concept analysis [9], such as context, formal concept lattice, or Galois connection. For the benefit of the reader not acquainted with the basics of the fuzzy extensions of the theory of formal concept analysis, we provide the preliminary notions below.

To begin with, the usual set of boolean values of classical logics (containing true and false), is generalized to the algebraic structure of complete residuated lattice, which allows to provide suitable extensions in a more abstract environment.

Definition 1 A complete residuated lattice is an algebra \( \langle L, \wedge, \vee, \otimes, \to, 0, 1 \rangle \)

1. \( \langle L, \wedge, \vee, 0, 1 \rangle \) is a lattice with least element 0 and greatest element 1,
2. \( \langle L, \otimes, 1 \rangle \) is a commutative monoid,
3. \( \otimes \) and \( \to \) are adjoint operators, i.e. \( a \otimes b \leq c \) if and only if \( a \leq b \to c \), for all \( a, b, c \in L \), where \( \leq \) is the lattice ordering generated from \( \wedge \) and \( \vee \).

Now, the natural extension of the notion of context is given below.

Definition 2 Let \( L \) be a complete residuated lattice, an \( L \)-fuzzy formal context is a triple \( \langle B, A, r \rangle \) consisting of a set of objects \( B \), a set of attributes \( A \) and an \( L \)-fuzzy binary relation \( r \), i.e. a mapping from \( r : B \times A \) to \( L \), which can be alternatively understood as an \( L \)-fuzzy subset of \( B \times A \).

We finalize the presentation of the preliminary definitions by introducing the \( L \)-fuzzy extension provided by Bělohlávek in [2], where we will use the notation \( Y^X \) to refer to the set of mappings from \( X \) to \( Y \).

Definition 3 Consider an \( L \)-fuzzy context \( \langle B, A, r \rangle \). A pair of mappings \( \uparrow : L_B \to L_A \) and \( \downarrow : L_A \to L_B \) is defined as follows:

\[
\uparrow f (a) = \bigwedge_{o \in B} (f(o) \to r(o, a)),
\]
\[
\downarrow g (o) = \bigwedge_{a \in A} (g(a) \to r(o, a)).
\]

for every \( f \in L_B \) and \( g \in L_A \).

Lemma 1 Let \( L \) be a complete residuated lattice, let \( r \in B \times A \) be an \( L \)-fuzzy relation between \( B \) and \( A \). Then the pair of operators \( \uparrow \) and \( \downarrow \) forms a Galois connection between \( \langle L_B, \subseteq \rangle \) and \( \langle L_A, \subseteq \rangle \).

This pair of mappings is said to be closed, in that they satisfy the equalities in Lemma 2 below.
Lemma 2 Under the conditions of Lemma 1, the following equalities hold for arbitrary \( f \in L^B \) and \( g \in L^A \):
\[
\uparrow f = \downarrow \downarrow f \quad \text{and} \quad \downarrow g = \uparrow \uparrow g.
\]

Definition 4 An \( L \)-fuzzy concept is a pair \( \langle f, g \rangle \) such that \( \uparrow f = g, \downarrow g = f \). The set of all \( L \)-fuzzy concepts associated to a fuzzy context \((B, A, r)\) will be denoted as \( L\text{-FCL}(B, A, r) \).

An ordering between \( L \)-fuzzy concepts is defined as follows: \( \langle f_1, g_1 \rangle \leq \langle f_2, g_2 \rangle \) if and only if \( f_1 \subseteq f_2 \) if and only if \( g_1 \geq g_2 \).

Theorem 1 The poset \((L\text{-FCL}(B, A, r), \leq)\) is a complete lattice where
\[
\bigwedge_{j \in J} \langle f_j, g_j \rangle = \left\langle \bigwedge_{j \in J} f_j, \uparrow \left( \bigwedge_{j \in J} f_j \right) \right\rangle,
\]
\[
\bigvee_{j \in J} \langle f_j, g_j \rangle = \left\langle \downarrow \left( \bigwedge_{j \in J} g_j \right), \bigwedge_{j \in J} g_j \right\rangle.
\]

We know recall the basic definitions and notations given in [16].

Definition 5 A correspondence from \( X \) to \( Y \) is a mapping \( f: X \to 2^Y \). Note that correspondences are also called set-valued or multiple-valued functions.

The transposed of a correspondence \( f: X \to 2^Y \) is a correspondence \( t^f: Y \to 2^X \) defined by \( t^f(y) = \{ x \mid y \in f(x) \} \).

The set \( \text{Cors}(X, Y) \) of all the correspondences from \( X \) to \( Y \) can be endowed of a poset structure by defining the ordering \( f_1 \leq f_2 \) as \( f_1(x) \subseteq f_2(x) \) for all \( x \in X \).

Definition 6 An \( L \)-correspondence from \( X \) to \( Y \) is a mapping \( \varphi: X \to L^Y \).

The transposed of an \( L \)-correspondence \( \varphi: X \to L^Y \) is an \( L \)-correspondence \( t^\varphi: Y \to X^L \) defined by \( t^\varphi(y)(x) = \varphi(x)(y) \).

The set \( \text{L-Cors}(X, Y) \) of all the \( L \)-correspondences from \( X \) to \( Y \) can be endowed of a poset structure by defining the ordering \( \varphi_1 \leq \varphi_2 \) as \( \varphi_1(x)(y) \leq \varphi_2(x)(y) \) for all \( x \in X \) and \( y \in Y \).

3 Chu correspondences

Let us recall the definition of Chu correspondence in the classical framework of crisp relations as contexts.

Definition 7 Let \( C_i = \langle B_i, A_i, R_i \rangle \) \((i = 1, 2)\) be crisp formal contexts. A pair \( f = (f_1, f_r) \) is called a correspondence from \( C_1 \) to \( C_2 \) if \( f_1 \) and \( f_r \), respectively, are correspondences from \( B_1 \) to \( B_2 \) and from \( A_2 \) to \( A_1 \).

A correspondence \( f \) from \( C_1 \) to \( C_2 \) is said to be a weak Chu correspondence if the following equality holds for every \( o_1 \in B_1 \) and \( a_2 \in A_2 \)
\[
\bigwedge_{y \in f_r(a_2)} R_1(o_1, y) = \bigwedge_{x \in f_1(o_1)} R_2(x, a_2)
\]
A weak Chu correspondence $f : C_1 \to C_2$ is called simply a **Chu correspondence** if the pair $f_1(o_1) \subseteq B_2$ and $f_r(a_2) \subseteq A_1$ is closed for every $o_1 \in B_1$ and $a_2 \in A_2$.

In the following we will concentrate in obtaining a suitable generalization of the previous definition to the framework $L$-fuzzy sets. To begin with, let us note that a given $L$-fuzzy context $r : B \times A \rightarrow L$ can be extended to the set $L$-fuzzy objects and attributes as follows. We will define a new mapping $\hat{r} : L^B \times L^A \rightarrow L$ such that for $f \in L^B$ and $g \in L^A$ we have

$$
\hat{r}(f, g) = \bigwedge_{o \in B, a \in A} (f(o) \otimes g(a) \rightarrow r(o, a)).
$$

This definition allows to provide a suitable generalization of Bělohlávček’s Galois connection as follows. Given a singleton $\{x\} \subseteq B$, consider the characteristic function of the singleton $\chi_x \in L^B$ as $\chi_x(x) = 1$ and $\chi_x(o) = 0$ for all $o \in B, o \neq x$. Then

$$
\hat{r}(\chi_x, g) = \bigwedge_{o \in B, a \in A} (\chi_x(o) \otimes g(a) \rightarrow r(o, a))
$$

$$
= \bigwedge_{o \in B, a \in A} (\chi_x(o) \otimes g(a) \rightarrow r(o, a) \land \bigwedge_{a \in A} (\chi_x(x) \otimes g(a) \rightarrow r(o, a))
$$

$$
= \bigwedge_{o \in B, a \in A} (0 \otimes g(a) \rightarrow r(o, a) \land \bigwedge_{a \in A} (1 \otimes g(a) \rightarrow r(x, a))
$$

$$
= \bigwedge_{a \in A} (g(a) \rightarrow r(x, a))
$$

which coincides with Bělohlávček’s definition, in which the element $x$ has been substituted by the characteristic function $\chi_x$. A similar result can be obtained by fixing a singleton in the set of attributes.

**Definition 8** Assume we have two formal contexts $C_i = (B_i, A_i, r_i), (i = 1, 2)$, then the pair $\varphi = (\varphi_l, \varphi_r)$ is called a **correspondence** from $C_1$ to $C_2$ if $\varphi_l$ and $\varphi_r$ are correspondences respectively from $B_1$ to $B_2$ and from $A_2$ to $A_1$ (i.e. $\varphi_l : B_1 \rightarrow L^{B_2}$ and $\varphi_r : A_2 \rightarrow L^{A_1}$). The correspondence $\varphi$ is called a **weak $L$-Chu correspondence** if for all $o_1 \in B_1$ and $a_2 \in A_2$ holds

$$
\hat{r}_1(\chi_{o_1}, \varphi_r(a_2)) = \bigwedge_{a_1 \in A_1} (\varphi_r(a_2)(a_1) \rightarrow r_1(o_1, a_1))
$$

$$
= \bigwedge_{a_2 \in B_2} (\varphi_l(o_1)(a_2) \rightarrow r_2(o_2, a_2)) = \hat{r}_2(\varphi_l(o_1), \chi_{a_2}).
$$

A weak Chu correspondence $\varphi$ is a **$L$-Chu correspondence** if the pair of mappings $\varphi_l(o_1)$ and $\varphi_r(a_2)$ is closed for all $o_1 \in B_1$ and $a_2 \in A_2$. We will denote the set of all Chu correspondences from $C_1$ to $C_2$ by $ChuCors(C_1, C_2)$. 

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We can check that this definition allows us to provide a suitable generalization of Mori’s definition of weak Chu correspondence and Chu correspondence as follows. Let us assume that we are in the classical case, that is, \( L = 2 \). Then

\[
\hat{r}_1(\chi_{o_1}, \varphi_r(a_2)) = \bigwedge_{a_1 \in A_1} (\varphi_r(a_2)(a_1) \rightarrow r_1(o_1, a_1))
\]

\[
= \bigwedge_{a_1 \in A_1} (\varphi_r(a_2)(a_1) \rightarrow r_1(o_1, a_1)) \land \bigwedge_{a_1 \in \varphi_r(a_2)} (\varphi_r(a_2)(a_1) \rightarrow r_1(o_1, a_1))
\]

\[
= 1 \land \bigwedge_{a_1 \in \varphi_r(a_2)} (\varphi_r(a_2)(a_1) \rightarrow r_1(o_1, a_1))
\]

\[
= \bigwedge_{a_1 \in \varphi_r(a_2)} (1 \rightarrow r_1(o_1, a_1)) = \bigwedge_{a_1 \in \varphi_r(a_2)} r_1(o_1, a_1)
\]

and similarly for \( \hat{r}_2 \).

4 Bonds

Here we will extend the classical definition of bond, as stated in [9] which is recalled below:

**Definition 9** A bond from a context \( C_1 = \langle B_1, A_1, R_1 \rangle \) to a context \( C_2 = \langle B_2, A_2, R_2 \rangle \) is a relation \( R_b \subseteq B_1 \times A_2 \) for which the following holds:

- \( \uparrow_b(o_1) = \{ a_2 \in A_2 : (o_1, a_2) \in R_b \} \) is an intent of \( C_2 \) for every \( o_1 \in B_1 \)
- \( \downarrow_b(a_2) = \{ o_1 \in B_1 : (o_1, a_2) \in R_b \} \) is an extent of \( C_1 \) for every \( a_2 \in A_2 \).

Now, we introduce our candidate for the \( L \)-fuzzy extension of the notion of bond.

**Definition 10** An \( L \)-bond between two formal contexts \( C_1 = \langle B_1, A_1, r_1 \rangle \) and \( C_2 = \langle B_2, A_2, r_2 \rangle \) is a correspondence \( b : B_1 \rightarrow L^{A_2} \) satisfying the condition that the pair \( b(o_1) \) and \( ^\dagger b(a_2) \) is closed for all \( o_1 \in B_1 \) and \( a_2 \in A_2 \). The set of all bonds from \( C_1 \) to \( C_2 \) is denoted as \( \text{Bonds}(C_1, C_2) \).

Every correspondence is a relation, thus an \( L \)-bond can be seen as a relation between \( B_1 \) and \( A_2 \). This certainly suggests a possible relationship between \( L \)-Chu correspondences and \( L \)-bonds.

**Definition 11**

- Let \( b : C_1 \rightarrow C_2 \) be an \( L \)-bond. We can define a correspondence \( \varphi_b : C_1 \rightarrow C_2 \) by

\[
\varphi_{bl}(o_1) = \downarrow_2 (b(o_1)) \in L^{B_2} \text{ for } o_1 \in B_1
\]

\[
\varphi_{br}(a_2) = \uparrow_1 (^\dagger b(a_2)) \in L^{A_1} \text{ for } a_2 \in A_2
\]
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• Conversely, consider an \( L \)-Chu correspondence \( \varphi \) from \( C_1 \) to \( C_2 \), and define another correspondence \( b_\varphi : B_1 \to L^{A_2} \) by

\[
b_\varphi(o_1) = \uparrow_2(\varphi_l(o_1))
\]

Proposition 1 With the definitions given above

1. \( \varphi_b \) is an \( L \)-Chu correspondence from \( C_1 \) to \( C_2 \).
2. \( b_\varphi \) is an \( L \)-bond from \( C_1 \) to \( C_2 \).

Proof: Both proofs follow as a result of more or less straightforward chains of computations. We will only include one of them.

1. Let \( o_1 \in B_1 \) and \( a_2 \in A_2 \). Then

\[
\hat{r}_2(\varphi_b(o_1), \chi_{a_2}) = \bigwedge_{o_2 \in B_2} (\varphi_b(o_1)(o_2) \to r_2(o_2, a_2))
\]

\[
= \bigwedge_{o_2 \in B_2} (\downarrow_2 b(o_1))(o_2) \to r_2(o_2, a_2))
\]

\[
= \uparrow_2 (\downarrow_2 b(o_1))(a_2) = b(o_1)(a_2) = \uparrow_1 (\uparrow_1 b(a_2))(o_1)
\]

\[
= \bigwedge_{a_1 \in A_1} (\uparrow_1 b(a_2))(a_1) \to r_1(o_1, a_1))
\]

\[
= \bigwedge_{a_1 \in A_1} (\varphi_{br}(a_2) \to r_1(o_1, a_1)) = \hat{r}_1(\chi_{o_1}, \varphi_{br}(a_2))
\]

\[\square\]

5 Relationship between Chu correspondences and Bonds

The previous proposition suggests a close relationship between the notions of \( L \)-Chu correspondence and \( L \)-bond between two formal contexts.

The existence of a possible isomorphism between these two type of structure have been checked by computer on several examples in a non-classical context, specifically on 3-valued logic. Now we present the results obtained in one specific cases and, for the purposes of this example we will consider a three-valued lattice \( L = \{\{1, 0, p\}, \leq\} \), with order \( \leq \) defined by \( 0 \leq p \leq 1 \).

Conjunction and implication are defined by

\[
a \otimes b = \min\{a, b\} \quad \text{and} \quad a \to b = \begin{cases} 1 & \text{if } a \leq b \\ b & \text{if } a > b \end{cases}
\]

In Table 1 we show the two 3-valued contexts we will be working with.
In Table 2 we show all the 3-bonds \( b \) between \( C_1 \) and \( C_2 \) (obtained by computer) together with all its associated 3-Chu correspondences \( (\varphi_{bl}, \varphi_{br}) \) computed by

\[
\varphi_{bl}(o_1) = \uparrow_2 (b(o_1)) \quad \text{and} \quad \varphi_{br}(a_2) = \downarrow_1 (t(b(a_2)))
\]

for all \( o_1 \in B_1 \) and \( a_2 \in A_2 \).

Then, in Table 3 all the 3-Chu correspondences \( (\varphi_l, \varphi_r) \) (obtained by computer) are presented, together with their associated 3-bonds computed by

\[
b_{\varphi_l}(o_1) = \uparrow_2 (\varphi_l(o_1)) \quad \text{or} \quad b_{\varphi_r} = t(b_{\varphi_l}) = \downarrow_1 (\varphi_r(a_2))
\]

for all \( o_1 \in B_1 \) and \( a_2 \in A_2 \).

The previous relationship between 3-Chu correspondences and 3-bonds holds as well in all the examples of particular 3-valued pairs of formal contexts we have considered. Therefore, it is likely that they are just instantiations of a general result, as in the classical case. Furthermore, we can define an ordering relation on the set of \( L \)-Chu correspondences between \( C_1 \) and \( C_2 \) as follows \( \varphi_1 \leq \varphi_2 \) if and only if for all \( (o_1, o_2) \in B_1 \times B_2 \) the following inequality holds \( \varphi_1(o_1)(o_2) \leq \varphi_2(o_1)(o_2) \). And a similar suitable ordering can be defined for \( L \)-bonds.

We will finalise this section by stating our main conjecture for future work.

**Conjecture 1** The correspondence which assigns to each Chu correspondence \( \varphi \) the bond \( b_{\varphi} \) is a bijection. Moreover, \( L\text{-ChuCors}(C_1, C_2) \) and \( L\text{-Bonds}(C_1, C_2) \) are isomorphic as ordered structures.

**References**


Table 2: Relationship between bonds and Chu correspondences between $C_1$ and $C_2$.

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Table 3: Relationship between Chu correspondences and bonds between $C_1$ and $C_2$.

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